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Stabilization of extra dimensions and the dimensionality of the observed space

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Abstract. We present a simple model for the late time stabilization of extra dimensions. The basic idea is that brane solutions wrapped around extra dimensions, which is allowed by string theory, will resist expansion due to their winding mode. The momentum modes in principle work in the opposite way. It is this interplay that leads to dynamical stabilization. We use the idea of democratic wrapping, where in a given decimation of extra dimensions, all possible winding cases are considered. To further simplify the study we assumed a symmetric decimation in which the total number of extra dimensions is taken to be Np where N can be called the order of the decimation. We also assumed that extra dimensions all have the topology of tori. We show that with these rather conservative assumptions, there exist solutions to the field equations in which the extra dimensions are stabilized and that the conditions do not depend on p. This fact means that there exists at least one solution to the asymmetric decimation case. If we denote the number of observed space dimensions (excluding time) by m, the condition for stabilization is $m \geq 3$ for pure Einstein gravity and $m \leq 3$ for dilaton gravity massaged by a string theory parameter, namely the dilaton coupling to branes.

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1 Introduction

String theory requires for its consistency extra dimensions that are bound to be very small *compared* to the size of observed dimensions. It is therefore of considerable importance to look for cosmological models in which this can be realized or at least not violently denied. String theory also allows for compact objects (branes) which could be wrapped around compact extra dimensions. These branes' winding modes will in general resist expansion in the same way as a rubber band would resist expansion of a balloon around which it is wrapped. The momentum (vibration) modes on the other hand would tend to enlarge the size of the brane. These two forces might in principle yield stabilization of the extra dimensions realized in a dynamical way. This idea is reminiscent of the Brandenberger–Vafa mechanism presented in [8]. In this letter we enlarge the mass of knowledge on a model which was in development during the past couple of years [1–7]. The interested reader should also check the literature on brane gas cosmology [9–26]. For further references one could check a recent review on the topic of brane gas cosmology [27]. These approaches are in essence different from the brane-world scenarios (see for example [28] for a review on the topic within the context of cosmology) which also make use of extra dimensions and try to reformulate the hierarchy problem in particle physics and other phenomena.

An interesting puzzle which could be addressed in any theory involving extra dimensions is the dimensionality of the observed space. This would mean to have an understanding of the question: why is the number of large dimensions three? In this paper we address the possibility that the requirement of stability (that is, no or a very small cosmological evolution present) of extra dimensions in general plus that of the dilaton in the context of string theory might give clues to the question. These stabilities are implied by positive observations on the zero or very small cosmological evolutions of the coupling constants which are supposed to change if the extra dimensions or the dilaton were evolving. The experimental bounds are rather stringent, so it makes sense to look for theories which incorporate absolute stability of extra dimensions and of the dilaton at least at the classical level. To this end we study the brane model we present in the context of pure Einstein gravity and then in the context of dilaton gravity. The bounds on the dimensionality m of observed space are different in both theories (we have $m \geq 3$ for pure Einstein gravity and $m \leq 3$ for dilaton gravity with the dilaton coupling to branes fixed by string theory) and agree only when $m = 3$, which also coincides with a stabilized dilaton. That is, assuming that the extra dimensions are stabilized one can think of Einstein gravity as emerging from dilaton gravity only when $m = 3$. On the other hand, taking

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the dilaton to be a constant at the outset is not necessarily compatible with the stability of extra dimensions for a general choice of m. This result is interesting of its own and we believe it is a rather solid result for explaining why the number of observed dimensions is three within the context of extra dimensions and string theory. Unfortunately this mechanism is not a dynamical one as the one advocated in [29] where the authors claim to explain the fact that $m = 3$ within the dynamics of string theory.[1]

Therefore we arrive at the concluding result: since experimentally the stability of the coupling constants requires the stability of extra dimensions and the dilaton, we have $m = 3$.

To make things simpler we assume that the extra dimensions (however they are partitioned as product spaces) are tori and hence flat and compact. In view of the lack of a general principle which would mandate a given wrapping pattern we use democratic winding as introduced in [5, 6]. To make things clearer let us proceed with an example: say the extra dimensions are partitioned into three tori of dimensions p, q and r. The democratic winding scheme requires us to allow for all possible windings and hence intersections. Namely the winding pattern will be as follows:

$$(p)qr \oplus p(q)r \oplus pq(r) \oplus (pq)r \oplus p(qr) \oplus q(rp) \oplus (pqr)\,.$$

Here parentheses mean that there is a brane of dimensionality equal to the sum of dimensions around which it wraps. For instance $(p)qr$ stands for a p dimensional brane wrapping only around the first partition, $q(rp)$ means there is a $p+r$ dimensional brane wrapping around the first and last partition and (pqr) is a $p+q+r$ dimensional brane covering all extra dimensions. For a general decimation pattern the model is complicated. However if we can show that there is a solution for a symmetric decimation in which the dimensionality of the partitionings are all the same (say p) and the total number of extra dimensions is Np and that this solution is independent of p, it will in general mean that there is at least one solution to the asymmetric partitioning case. The ideas of winding democracy and symmetric partitioning were introduced in [5] for pure Einstein gravity and in [6] for dilaton gravity. It was shown that the stabilization conditions are p independent in both cases. However, in [5] brane momentum modes were not considered, and there remained an N dependence on the stabilization conditions, whereas in [6] it was shown that the results really do not depend on N if one also considers momentum modes. The main idea of this paper is first to add momentum modes to the pure Einstein gravity case and coherently study the two models in such a way as to finally contrast the conditions imposed on the dimensionality of observed space by the requirement of stabilization of the extra dimensions.

[1] After the present manuscript appeared Randall and Karch [30] came up with another variant [29] of a theory of why we live in three dimensions. The present manuscript is not along the same line of reasoning; we simply show that $m = 3$ is the only possibility if one also requires the stability of extra dimensions and the dilaton. We simply assume that "some" number of dimensions are somehow singled out at the outset but keep m as a parameter to be fixed later on.

2 General formalism

Since we are interested in a cosmological model we take our metric to be

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + \mathrm{e}^{2B(t)}\,\mathrm{d}x^2 + \sum_i \mathrm{e}^{2C_i(t)}\,\mathrm{d}y_i^2\,. \qquad (1)$$

Here B stands for the scale factor of the observed space with dimensionality m. The C_i are the scale factors of the extra dimensions. There are N such factors, each corresponding to p dimensional tori. The total dimensionality of space-time is $d = m + 1 + Np$. Because of the symmetric decimation pattern we can take all C_i to behave the same way.

We will also assume that the branes are distributed as a continuous gas with respect to the directions they are not wrapping. This makes it possible to use dust-like energy-momentum tensors. That is, for any energy-momentum source λ we assume the following form for the energy density which is found by the conservation requirement of the energy-momentum tensor for that particular source[2]:

$$\rho_\lambda = \rho_\lambda^0 \exp\left[-\left(1+\omega_B^\lambda\right) mB - \sum_i \left(1+\omega_{C_i}^\lambda\right) pC_i\right]. \qquad (2)$$

Here the ω are the pressure coefficients. It was shown in [1–4] that if we consider a homogeneous gas of branes the winding mode of a p-brane will yield a conserved dust-like energy-momentum tensor with pressure coefficient -1 along the winding directions and 0 for the other ones. For example the energy densities of the winding modes of a p-brane, a $2p$-brane and an Np-brane will respectively be

$$\rho_p = \rho_p^0 \mathrm{e}^{-mB} \mathrm{e}^{-(N-1)pC}\,,$$
$$\rho_{2p} = \rho_{2p}^0 \mathrm{e}^{-mB} \mathrm{e}^{-(N-2)pC}\,,$$
$$\rho_{Np} = \rho_{Np}^0 \mathrm{e}^{-mB}\,.$$

For the momentum modes the pressure coefficient of a p-brane can be taken to be $1/p$ along the winding directions and vanishing for the rest [2–4]. So the momentum mode energy densities for the above list will be

$$\tilde\rho_p = \tilde\rho_p^0 \mathrm{e}^{-mB} \mathrm{e}^{-C-NpC}\,,$$
$$\tilde\rho_{2p} = \tilde\rho_{2p}^0 \mathrm{e}^{-mB} \mathrm{e}^{-C-NpC}\,,$$
$$\tilde\rho_{Np} = \tilde\rho_{Np}^0 \mathrm{e}^{-mB} \mathrm{e}^{-C-NpC}\,.$$

This very simple behaviour of the momentum modes will be a crucial ingredient in proving stabilization. We

[2] So the total energy momentum tensor will be a sum of such separately conserved contributions.

have also adopted a convention for the initial values of the energy densities: the ρ^0 will correspond to winding modes and the $\tilde{\rho}^0$ will correspond to momentum modes.

2.1 Pure Einstein gravity

The field equations for pure Einstein gravity are

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa^2 T_{\mu\nu} \,. \tag{3}$$

With our assumptions the equations of motion for the scale factors can be cast in the following form (we set $\kappa^2 = 1$):

$$\dot{A}^2 = m\dot{B}^2 + \sum_i p_i \dot{C_i}^2 + 2\rho \,, \tag{4a}$$

$$\ddot{B} + \dot{A}\dot{B} = T_{\hat{b}\hat{b}} - \frac{1}{d-2} T \,, \tag{4b}$$

$$\ddot{C}_i + \dot{A}\dot{C}_i = T_{\hat{c}_i\hat{c}_i} - \frac{1}{d-2} T \,, \tag{4c}$$

$$A \equiv mB + \sum_i p_i C_i \,. \tag{4d}$$

The hatted indices refer to the orthonormal co-ordinates. Also ρ represents the total energy density and $T_{\hat{\mu}\hat{\nu}}$ are the components of the total energy-momentum tensor while T is its trace.

Stabilization of the extra dimensions will imply

$$T_{\hat{c}_i\hat{c}_i} - \frac{1}{d-2} T = 0 \,. \tag{5}$$

Considering all the energy-momentum tensors in a democratic winding scheme will after straightforward algebra yield the following :

$$\mathrm{e}^{-mB-NpC} \left[\frac{1}{p} \alpha X^{-1} + \frac{1}{d-2} \sum_{k=1}^{N} \beta_k \zeta_k X^{kp} \right] = 0 \,, \tag{6}$$

with $X \equiv \mathrm{e}^C$, the scale factor of the extra dimensions, and

$$\alpha = \sum_{i=1}^{N-1} \tilde{\rho}^0_{ip} + \tilde{\rho}^0_{Np}/N \,, \tag{7a}$$

$$\beta_k = \rho^0_{kp} \,, \tag{7b}$$

$$\beta_N = \rho^0_{Np}/N \,, \tag{7c}$$

$$\zeta_k = N - k(m-1) \,. \tag{7d}$$

The difference in the definition of β_N is due to the fact that there is only one brane wrapping over all extra dimensions.

To show that there is stabilization we have to find positive solutions to the polynomial in (6). In order to study this we can use Descartes' sign rule which states that the positive roots of a polynomial is either equal to the number of sign changes s of the coefficients or less than s by a multiple of 2. Since α and β_k are all positive numbers the sign changes will be ruled by ζ_k. But the ζ_k are monotonically decreasing by k for a given m, so there can only be one sign change in the polynomial (6) and hence only one positive root exists. The worst case therefore is given by $\zeta_N \geq 0$, which would mandate every term to be positive. This means that to have a sign change we need $m \geq 2$; however, for $m = 2$, ζ_N is zero and $\zeta_{N-1} > 0$.

Consequently the real constraint to have a solution is

$$m \geq 3 \,. \tag{8}$$

This result does not depend on N or on p, so there must be at least one solution for stabilization even in the case (very difficult to analyze) for which the decimation of extra dimensions is not symmetric. It can also be shown that with these stabilization conditions the observed space expands with the same power-law $(2/m)$ as pressureless dust. This is expected since all the brane energy-momentum tensors are pressureless dust for the observed space.

2.2 Dilaton gravity

We can take the action in the presence of a dilaton field ϕ coupled to matter to be [4]

$$S = \frac{1}{\kappa^2} \int \mathrm{d}x^d \sqrt{-g} \mathrm{e}^{-2\phi} \left[R + 4(\nabla\phi)^2 + \mathrm{e}^{a\phi} \mathcal{L}_m \right] \,. \tag{9}$$

If $\mathcal{L}_m$ takes the form of a Lagrangian yielding a dust-like energy-momentum tensor the field equations are [4]

$$R_{\mu\nu} + 2\nabla_\mu\nabla_\nu\phi = \mathrm{e}^{a\phi} \left[T_{\mu\nu} - \left(\frac{a-2}{2} \right) \rho g_{\mu\nu} \right] \,, \tag{10a}$$

$$R + 4\nabla^2\phi - 4(\nabla\phi)^2 = -(a-2)\mathrm{e}^{a\phi}\rho \,, \tag{10b}$$

which in turn will give the following (we set $\kappa^2 = 1$):

$$\ddot{B} = -k\dot{B} + \mathrm{e}^{a\phi} \left[T_{\hat{b}\hat{b}} - \tau\rho \right] \,, \tag{11a}$$

$$\ddot{C}_i + k\dot{C}_i = \mathrm{e}^{a\phi} \left[T_{\hat{c}_i\hat{c}_i} - \tau\rho \right] \,, \tag{11b}$$

$$\ddot{\phi} = -k\dot{\phi} + \frac{1}{2} \mathrm{e}^{a\phi} \left[T - (d-2)\tau\rho \right] \,, \tag{11c}$$

$$k^2 = m\dot{B}^2 + \sum_i p_i \dot{C_i}^2 + 2\mathrm{e}^{a\phi}\rho \,, \tag{11d}$$

$$k \equiv m\dot{B} + \sum_i p_i \dot{C}_i - 2\dot{\phi} \,, \tag{11e}$$

where $\tau = (a-2)/2$. The stabilization condition is

$$\mathrm{e}^{a\phi} \left[T_{\hat{c}_i\hat{c}_i} - \tau\rho \right] = 0 \,. \tag{12}$$

Similarly to the previous subsection, after considering all the energy-momentum contributions, this will yield the following:

$$\mathrm{e}^{a\phi-mB-NpC} \left[\left(\frac{1}{p} - \tau N \right) \alpha X^{-1} - \sum_{k=1}^{N} \beta_k \xi_k X^{kp} \right] = 0 \,. \tag{13}$$

Here α and β_k are the same parameters as in the previous subsection, as in (7), and $\xi_k = k + \tau N$. The discussion for a solution is very similar to the previous section. There can only be one sign change in the polynomial (13) due to the linear change in ξ_k. It is easy to show that solutions will exist for

$$-1<\tau<\frac{1}{Np}\Longrightarrow -1<\tau<\frac{1}{d-m-1}\,. \tag{14}$$

Again the constraints do not depend on N or p, and this means that there should at least be one solution to the stabilization conditions for asymmetric decimations.

Furthermore in [6] it has been shown that the observed space's scale factor and the dilaton evolve according to a power-law ansatz,

$$B(t)\sim\beta\ln t\,, \tag{15a}$$
$$\phi(t)\sim\varphi\ln(t)\,, \tag{15b}$$

with

$$\beta=-\frac{2\tau}{1+(m-1)\tau^2}\,, \tag{16a}$$
$$\varphi=\frac{-2+m\beta}{2(1+\tau)}=-\frac{1+\tau(m-1)}{1+(m-1)\tau^2}\,. \tag{16b}$$

Since we want to use these as late time cosmology solutions we would like to have the scale of the observed space expanding. Also to not enter the strong coupling regime of string theory at late times we would like to have a decreasing (or stable) dilaton solution.[3] Thus we want $\beta>0$ and $\varphi\le 0$ in the equation above. These further requirements will alter the stabilization conditions in the following way:

$$-\frac{1}{m-1}\le\tau<0\,. \tag{17}$$

In string theory $\tau=-1/2$ for Dp-branes. This in turn means that

$$m\le 3\,. \tag{18}$$

3 Stability

The pure Einstein and dilaton gravity cases share a common property for the evolution of the extra dimensions. The equations governing the behaviour of the scale factors of extra dimensions is always in the following form:

$$\ddot{C}=-f(t,\dot{C})\dot{C}+g(t)X^{-Np-1}P(X)\,, \tag{19}$$

with again $X=\mathrm{e}^C$. The polynomial $P(X)$ has a single positive root. The general structure of this polynomial is as follows:

$$P(X)=1+a_0X+a_1X^{p+1}+\cdots+a_NX^{Np+1}\,, \tag{20}$$

where the constant term comes from the collective momentum modes, the linear term comes from ordinary pressureless matter living in the observed space (and hence has zero pressure coefficients everywhere), and the terms involving powers of p come from the winding modes. The condition for stabilization is that after/before the kth term all a_k are negative/positive. Therefore, invoking Descartes' rule again, we would have unique solutions for the vanishing of the derivatives up to the $(k-1)$th order. Thus $P(X)$ increases starting from $X=0$ and starts decreasing after the unique solution to $P'(X)=0$, until it reaches the unique stabilization point $P(X_0)=0$.

On the other hand the function $g(t)$ is always positive.[4] As it stands the equations describe the motion of a particle under the influence of a position dependent force and a velocity dependent friction/driving force f. As one could guess if $f<0$, the stabilization might be jeopardized – and it is, this fact having been numerically substantiated in [4]: if $f<0$ there is a singularity in the field equations in finite proper time. If one takes a good look at the field equations in either pure Einstein or dilaton gravity the sign of f is a constant of motion.[5] We therefore should consider the $f>0$ case.

As for the position dependent force we can look for the potential that gives rise to it

$$-\frac{\mathrm{d}V_{\text{eff}}}{\mathrm{d}X}=X^{-Np-1}P(X)\,. \tag{21}$$

The general form of $P(X)$ and $V_{\text{eff}}(X)$ are represented in Figs. 1 and 2. The potential has a unique minimum, and therefore the stabilization solutions are truly stable since $f>0$ will just bring in the friction. The effective potential has a very strong repulsive core near the small X region in the form inverse powers of X, the strongest one coming from the collective momentum modes. The large X behaviour is dominated by the winding mode of the largest brane in the system; in a democratic wrapping scheme this would be a term linear in X and would come from the Np-brane that wraps the entire decimation. These results are very plausible: the momentum modes are the ones that resist the contraction most, and the largest brane's winding mode is the one that most resists expansion.

We thus have shown that the stabilization solution is a future attractive point given $f(t=0)>0$ and the internal dimensions will evolve to that point no matter what the initial conditions are.[6] It has also recently been shown by Kaya [7] that the stabilization point is also dynamically stable against cosmological perturbations of the metric.

[3] The requirement for a decreasing or stable dilaton is the phenomenologically favored situation. One could in principle look for the increasing dilaton case and this can prove to be of interest in another context. We will not pursue this idea here.

[4] Since it is just a positive factor times e^{-mB}.

[5] This follows from the definitions of $\dot{A}$ and k in (4a), (4d) and (11d), (11e) respectively.

[6] The stability analysis we have exposed here is only valid in the classical regime. The ultimate analysis should have included the effects of quantum fluctuations, which we do not pursue here since it would require all-out use of string theory. We advocate on the other hand that the classical stability is a strong argument, since it actually becomes more and more valid at later times when string theory becomes less and less important.

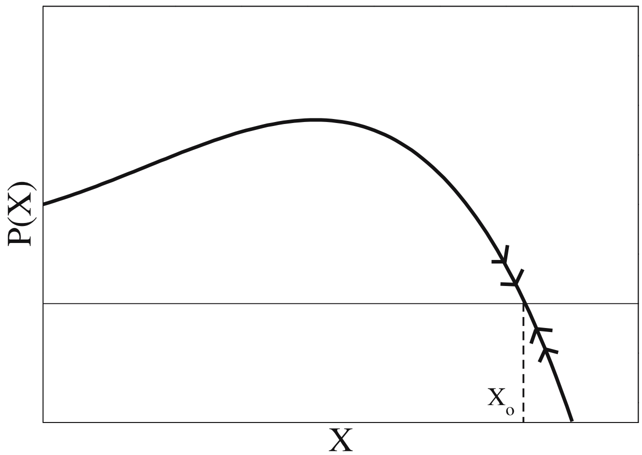

Fig. 1. The generic form of the stabilization polynomial in (20)

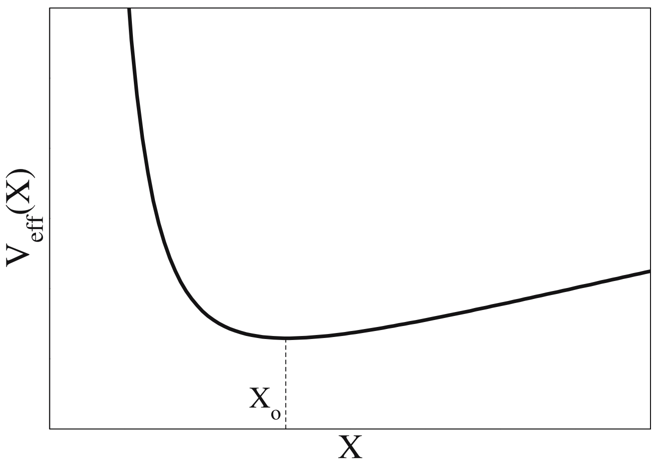

Fig. 2. The generic form of the effective potential defined in (21)

4 Conclusions and discussions

The analysis of the previous chapter can be summarized as follows. The late time stabilization of extra dimensions by the dynamics of Dp-branes require

$$m \geq 3 \quad \text{pure Einstein gravity}\,.$$
$$m \leq 3 \quad \text{dilaton gravity (string theory input } a = 1)\,.$$

The only case in which they would both agree is the experimentally observed case, $m = 3$. In this case the dilaton is also stabilized and the observed space expands as the ordinary pressureless dust solution with power $2/3$. The fact that the dilaton stabilizes means that the Einstein frame and the string frame are the same in the far future. One of the reasons why two different models yield different regimes for stabilization is that one should not really think that pure Einstein gravity can be obtained from dilaton gravity by setting the dilaton to a constant, because the evolution equation for the dilaton is not necessarily satisfied identically for every parameter of the system. However, one can think of obtaining pure Einstein gravity from dilaton gravity by setting the dilaton to a constant when $m = 3$.

Since one would like to recover Einstein gravity for low energies it seems that $m = 3$ is mandated by stabilization of extra dimensions.

Although we have confined the present study to the symmetric decimation of the extra dimensions (each one having dimensionality p), the fact that the stabilization condition does not depend on p means that there should be at least one solution to the stabilization equations in the asymmetric decimation case. So the mechanism is generic.

It is also rather interesting that in the dilaton case we have found $m \leq 3$ without requiring that in the early universe p-branes with $p > 2$ are annihilated as one would argue in the case of the Brandenberger–Vafa mechanism. If one would like to apply this constraint of the Brandenberger-Vafa mechanism to the model of this work no part of a decimation can have $p > 2$, and there is simply no solution for stabilization in this case.

One important point to mention is the possibility to allow for internal curvature for the extra dimensions. An in-depth study is not within the scope of this manuscript; however, a simple analysis shows that the internal curvature frustrates the mechanism we presented. The reason is that the curvature will bring about a factor of $2k\mathrm{e}^{-2C}$ to the stabilization polynomials, and, since this term has no overall e^{-mB} factor, stabilization cannot be achieved as we pointed out. On the other hand this can also be interpreted as a hint that extra dimensions have no curvature.

One immediate extension of the work presented here is the addition of pressureless dust (galaxies, quasars etc.) in the observed dimensions. Since this form of matter will bring about a positive term with a factor e^{-mB} to the stabilization polynomial, the mechanism we have presented will work out as well, at least for Einstein gravity. For dilaton gravity on the other hand one will have to know the coupling a_{matter} of the dilaton to ordinary matter, and this is not known. However, if the dilaton stabilization occurs before matter domination, nothing will change.

Among other important extensions one has to study is the study of the model during the radiation and the early inflationary eras. Early inflation tends to grow extra dimensions as well as the observed dimensions unless one introduces rather contrived and unmotivated cancellation mechanisms. Consequently during radiation a shrinking of the extra dimensions has to occur in order to have them as small as the experimental bounds mandate. A preliminary analysis shows this to be true, but we leave an extensive study for future work. This bounce back of the size of the extra dimensions can be very useful if used together with the bounds on the change of the fine structure constant coming from primordial nucleosynthesis. There could also be interesting avenues if one includes the present accelaration of the universe.

4.1 Note added

One important assumption we have made in this manuscript is that the gas of branes wrapping around the extra dimensions is dilute and that it is non-interacting. This assumption manifests itself in the derivation of the pressure coefficients of the winding and momentum modes [1–4] as they are obtained from a non-interacting action and hence only kinematical. The dilute, non-interacting, gas assumption also lets us take the full energy-momentum tensor as a sum of the contribution of different branes. The thermodynamics of the brane gas can in principle alter the pressure coefficients (see for example [31]), but in a perturbative scheme these effects should come as corrections containing higher order string effects. Furthermore it is very plausible that the dilute gas forms later in cosmological evolution when the size of the large dimensions grew much bigger than the size of extra dimensions. With this in mind we recall that the main idea and the result of this paper is the interplay between the stabilization of extra dimensions, stabilization of the dilaton and the dimensionality of the observed space. One could argue that correction effects should not alter the bound on m, since it is an integer. In this paper we have only taken one parameter from string theory: the dilaton coupling to branes $a=1$.

Another point related to string theory corrections is at the Lagrangian level. Higher order string corrections will in principle yield extra terms in the Lagrangian and in a low energy point of view they will be manifested as higher order gravity actions. After this paper appeared, Borunda and Boubekeur studied the effect of alpha' corrections in brane gas cosmology. We refer the interested reader to their work [32].

Appendix

In this appendix for didactical purposes we would like to expose the simpler case of $N=1$. That is, we assume that the extra dimensions are lumped in a p dimensional torus. The energy densities for the brane winding and momentum modes will be given by

$$\rho_p = \rho_p^0 \mathrm{e}^{-mB}\,, \tag{A.1a}$$
$$\tilde{\rho}_p = \tilde{\rho}_p^0 \mathrm{e}^{-mB-(1+p)C}\,. \tag{A.1b}$$

Pure Einstein gravity

The equation for the evolution of the extra dimensions will be

$$\ddot{C} + \dot{A}\dot{C} = -\frac{m-2}{d-2}\rho_p + \frac{1}{p}\tilde{\rho}_p\,. \tag{A.2}$$

It is obvious from the equation above that in order to have $\dot{C}=0$ and $\ddot{C}=0$ we need to have $m \geq 3$. The remaining equations for B are

$$m(m-1)\dot{B}^2 = 2\rho_p + 2\tilde{\rho}_p\,, \tag{A.3a}$$
$$\ddot{B} + m\dot{B}^2 = \frac{1+p}{d-2}\rho_p\,. \tag{A.3b}$$

Assuming a power-law ansatz of the form $B(t) = \beta\ln(t) + B_0$ will yield

$$\beta = \frac{2}{m}\,, \tag{A.4}$$
$$\mathrm{e}^{-mB_0} = 2\frac{d-2}{\rho_p^0 m(1+p)}\,. \tag{A.5}$$

Dilaton gravity

The case $N=1$ has actually been studied in detail by Arapoglu and Kaya [4]. Quoting verbatim from their paper (where they took $a=1$ from the start) they use the following ansatz:

$$\phi = \phi_1\ln(t) + \phi_0\,, \tag{A.6a}$$
$$B = b_1\ln(t)\,, \tag{A.6b}$$
$$C = C_0\,, \tag{A.6c}$$

and we find that using the evolution equations (11a), (11b) and (11c) we get

$$b_1 = \frac{4}{m+3}, \quad \phi_1 = \frac{2(m-3)}{m+3}\,, \tag{A.7a}$$
$$\mathrm{e}^{\phi_0} = \frac{4(p+2)p}{T_w(p+1)(m+3)^2}\,, \tag{A.7b}$$
$$\mathrm{e}^{(p+1)C_0} = \frac{(p+2)T_m}{pT_w}\,, \tag{A.7c}$$

where $T_w = \rho_0$ and $T_m = \tilde{\rho}_0$. With these the constraint equation (11d) is identically satisfied. As can be checked the values for ϕ_1 and b_1 are in accord with β in (16a) and φ in (16b) for $a=1$, meaning $\tau = -1/2$.

Now it is clear that to have a decreasing dilaton, and so as to not enter the strong coupling regime of string theory in the far future one has to have $m \leq 3$.

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